

AN ALGORITHMIC IMPLEMENTATION OF THE π FUNCTION BASED ON A NEW SIEVE

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ABSTRACT. In this paper we propose an algorithm that correctly discards a set of numbers (from a previously defined sieve) with an interval of integers. Leopoldo's Theorem states that the remaining integer numbers will generate and count the complete list of primes of absolute value greater than 3 in the interval of interest. This algorithm avoids the problem of generating large lists of numbers, and can be used to compute (even in parallel) the prime counting function $\pi(h)$.

Keywords: Prime numbers, sieve, prime counting function, prime counting algorithm.

1. INTRODUCTION

An old problem in mathematics is the way to compute the amount of prime numbers less or equal to a given value h [2, p. 347]. This function is known as $\pi(h)$ [4]. The preeminent method for such task since the 3rd century BC was the sieve of Eratosthenes. From then on there were no great advances on the subject until Gauss in 1863 [2, p. 352]. His work allowed more advances [3, 6] and then boosted by the growth of calculation power in the 20th century. Recent implementations [6] require several sophisticated computational strategies.

One of the major difficulties in solving this issue -or may be the only one- is the question of how prime numbers are distributed among integers. Mathematicians and other scientists have carried great efforts to find some general formula and simple prime generating formula. Such formula is still elusive. Some formulas have been proposed, but they are valid only in certain cases, such as Mersenne numbers, and others. The key to find a simple formula that would allow us determine all prime numbers, should begin demonstrating that all numbers obey a general logic order, from there on it could be possible to find a sequence that would permit us to establish such formula. In other words it is about establishing a new sieve of universal validity. This basic essential aspect was the one which guided L. Euler in 1770 to express “*Mathematicians have tried in vain to discover some order in the sequence of prime numbers but we have every reason to believe that there are some mysteries which the human mind will never penetrate*”.

On the other side, the efforts of mathematicians like Mersenne, Fermat, and many others from Ancient Greece up to our days, to introduce certain logic in the calculus of some prime numbers with simple arithmetic operations became useless due to the observation made by Gauss [2] himself, who stated that the amount of prime numbers less than h is near the value of the integral:

$$\text{Li}(h) = \int_2^h \frac{dt}{\ln(t)}$$

This observation allowed Gauss to introduce the function $\pi(h)$ together with a probabilistic argument in the studies about the distribution of primes. In fact the function $\pi(h)$ is related with the *Fehlerintegral* or Error function (Erf), introduced by Gauss to analyze the uncertainties that appear in experimental measurements. It is to be noted, that in experimental sciences like Physics or Biology, the scientist needs to justify that the results of a certain experiment can be treated inside the gaussian frame. This a priori it does not seem to happen in the mathematics area, in particular in Number Theory, where Axioms play a fundamental role. Nevertheless, in the last years a lot of works have merged regarding the issue of the location of prime numbers, whether and if primes are located in a random way or if they have a totally determined structure. The papers that were published in this area are various and interesting (see references [9, 10, 11, 13, 16, 14]). These results are not completely satisfactory, therefore “the human mind” –according to L. Euler– will continue the attempt to penetrate the mysteries of the apparent unbreakable tower of primes. In particular, about this matter, Terence Tao [8] proposes the existence of certain dichotomy between structure and randomness among primes, and tries to separate the structured component (or correlated) from the “pseudorandom” component (or decorrelated). Anyway, the existence of a distribution of prime numbers affected by the dichotomy of structured-randomness implies that the structured behavior or low complexity behavior can be determined by any general formula, meanwhile in the high complexity behavior we could recognize at least two aspects: i) A totally randomness aspect or decorrelated, and the other, ii) A chaotic one, which obeys some kind of logic formulation. In our opinion any well known prime number allows us to describe a sieve, which can be enlarged as far as new primes are verified as such numbers [1]. However we consider that the unknown prime numbers, are not undefined nor undetermined. That means that all of them can be defined and determined by a general and unique algorithm, logically determined and located sequentially among integers [5].

Returning to our primary purpose of implementing an algorithm to calculate the value of the $\pi(h)$ function for a given h , in this paper we will study this problem defining a new sieve whose properties permit an elementary study of $\pi(h)$, together with the possibility of finding its value on a given interval. The basis of this sieve stands on an infinite matrix presented in [1], which has the property of generating all indexes that lead to non prime numbers according to the form $6n + 1$ or $6n - 1$, a similar approach can be found in [7], but with a different implementation.

The paper is organized as follow. In Section 2. we reviewed the properties of the form $6n + 1$, being $n = 0, 1, 2, \dots$, which is the base of our research. Section 3 is devoted to our proposal of a sieve to define and compute all prime numbers larger than 3. In Sections 4 and 5 we present the Leopoldo’s Theorem and the possibility to compute $\pi(h)$ function according with the Λ algorithm that we are introducing. Section 6 and 7 are devoted to our actual results, comments on future research work and conclusions.

2. ABOUT THE FORM $6n + 1$

In [1] we reviewed some well known properties of numbers:

Theorem 2.1. *Every prime number of absolute value greater than 3 can be written in the form $6n + 1$ or $6n - 1$.*

Proof. Let's see the equivalences modulo 6. Suppose q prime.

- 1) If $q \equiv 0 \pmod 6 \Rightarrow q = 6n \Rightarrow 6 \mid q$, ABS.
- 2) If $q \equiv 1 \pmod 6 \Rightarrow q = 6n + 1$, which is not impossible since $7 = 6 + 1$ is a prime.
- 3) If $q \equiv 2 \pmod 6 \Rightarrow q = 6n + 2 = 2(3n + 1) \Rightarrow 2 \mid q$, ABS.
- 4) If $q \equiv 3 \pmod 6 \Rightarrow q = 6n + 3 = 3(2n + 1) \Rightarrow 3 \mid q$, ABS.
- 5) If $q \equiv 4 \pmod 6 \Rightarrow q = 6n + 4 = 2(3n + 2) \Rightarrow 2 \mid q$, ABS.
- 6) If $q \equiv 5 \pmod 6 \Rightarrow q = 6n + 5 = 6n + 6 - 1 = 6(n + 1) - 1$, which is not impossible since with $n = 1$ this gives 11, a prime.

Then we introduce some definitions: □

Definition 2.2. The α class of integer numbers [5] is the set

$$(2.1) \quad \alpha = \{x \in \mathbb{Z} / x = 6n + 1, n \in \mathbb{Z}\}$$

Definition 2.3. The β class of integer numbers [5] is the set

$$(2.2) \quad \beta = \{x \in \mathbb{Z} / x = 6n - 1, n \in \mathbb{Z}\}$$

The different values of relations in (2.1) and (2.2) are shown in Table 1.

n	$\beta_n = 6n - 1$	$\alpha_n = 6n + 1$
\vdots	\vdots	\vdots
-5	-31 *	-29 *
-4	-25	-23 *
-3	-19 *	-17 *
-2	-13 *	-11 *
-1	-7 *	-5 *
0	-1	1
1	5 *	7 *
2	11 *	13 *
3	17 *	19 *
4	23 *	25
5	29 *	31 *
\vdots	\vdots	\vdots

TABLE 1. Values of $6n - 1$ and $6n + 1$ for several n . With (*) we mark prime numbers.

Strictly speaking, a “complete” list of all prime numbers of absolute value greater than 3 ($\{\dots, -7, -5, 5, 7, \dots\}$) is the list of primes from both classes, α and β .

We now state a property given in [5] and [1]:

Theorem 2.4. *Every prime number of absolute value greater than 3 (except for the sign) is generated by $6n + 1$, with n integer.*

Proof. We must prove the equivalence (except for the sign) between both families given in Theorem 2.1.

Let be $f_\alpha(n) = 6n + 1$ and $f_\beta(n) = 6n - 1$, we must now prove that $f_\alpha(-n) = -f_\beta(n)$. Indeed:

$$f_\alpha(-n) = 6(-n) + 1 = -6n + 1 = -(6n - 1) = -f_\beta(n)$$

□

Definition 2.5. We define the set of integer numbers G_α :

$$(2.3) \quad G_\alpha = \{g \in \mathbb{Z}/6g + 1 \text{ is a prime}\}$$

This means, G_α is the set of *all* numbers that (except for the sign) generate *all* primes of absolute value greater than 3 by the relationship (2.1).

In other words, this result is perfectly logic because it is based on:

- i) Peano's Axioma applied to both sequences of numbers α and β starting in 1 and 5, respectively, taking into account that the next number in those sequences are defined as $1 + 6$, and $5 + 6$, respectively, and so on and so forth; and
- ii) Remark (3.5) which follows from Theorem 3.4., represented by the Table of Products 4. This table states that products between numbers belonging to the α sequence and products between numbers belonging to the β sequence, results in numbers of Class α ; while the crossed products between numbers of the sequence α with numbers of the sequence β , results in numbers of Class β . Besides, note that crossed products between numbers belonging to the six classes we presented in Theorem 2.1. never result in numbers belonging to classes α or β . Table 2 shows the results of these products from $n = 0$ to $n = 10$; remember, prime numbers greater than 3 are only allocated in classes α or β .

n	$6n + 1(\alpha)$	$6n + 2$	$6n + 3$	$6n + 4$	$6n + 5(\beta)$	$6n + 6$
0	1	2	3	$4 = 2 \times 2$	5	$6 = 2 \times 3$
1	7	$8 = 2 \times 4$	$9 = 3 \times 3$	$10 = 2 \times 5$	11	$12 = 2 \times 6$
2	13	$14 = 2 \times 7$	$15 = 3 \times 5$	$16 = 2 \times 8$	17	$18 = 2 \times 9$
3	19	$20 = 2 \times 10$	$21 = 3 \times 7$	$22 = 2 \times 11$	23	$24 = 2 \times 12$
4	$25 = 5 \times 5$	$26 = 2 \times 13$	$27 = 3 \times 9$	$28 = 2 \times 14$	29	$30 = 2 \times 15$
5	31	$32 = 2 \times 16$	$33 = 3 \times 11$	$34 = 2 \times 17$	$35 = 5 \times 7$	$36 = 2 \times 18$
6	37	$38 = 2 \times 19$	$39 = 3 \times 13$	$40 = 2 \times 20$	41	$42 = 2 \times 21$
7	43	$44 = 2 \times 22$	$45 = 3 \times 15$	$46 = 2 \times 23$	47	$48 = 2 \times 24$
8	$49 = 7 \times 7$	$50 = 2 \times 25$	$51 = 3 \times 17$	$52 = 2 \times 26$	53	$54 = 2 \times 27$
9	$55 = 5 \times 11$	$56 = 2 \times 28$	$57 = 3 \times 19$	$58 = 2 \times 29$	59	$60 = 2 \times 30$
10	61	$62 = 2 \times 31$	$63 = 3 \times 21$	$64 = 2 \times 32$	$65 = 5 \times 13$	$66 = 2 \times 33$
...

TABLE 2. Numbers belonging to the six classes we divided natural numbers from $n = 0$ to $n = 10$. Prime numbers are shown in bold letters.

Definition 2.6.

3. A PROPOSED NEW SIEVE

Definition 3.1. Let A be an infinite matrix whose element $a(i, j)$ ¹ is

$$(3.1) \quad a(i, j) = i + j(6i + 1)$$

where $i, j \in \mathbb{Z}$.

In Table 3 we show the elements in the central part of the A matrix. Note that axis numbers also match this representation.

				\vdots					
	-96	-71	-46	-21	4	29	54	79	104
	-73	-54	-35	-16	3	22	41	60	
	-50	-37	-24	-11	2	15	28		
	-27	-20	-13	-6	1	8			
\dots	-4	-3	-2	-1	0	1	2	3	\dots
	19	14	9	4	-1				
	42	31	20		-2				
	65	48			-3				
	88				-4				
					\vdots				

TABLE 3. Elements of the central portion of A

3.1. Properties.

Theorem 3.2. A is symmetrical.

Proof. A simple expansion shows that

$$a(i, j) = i + j(6i + 1) = i + 6ij + j = j + i(6j + 1) = a(j, i)$$

□

Definition 3.3. Let \tilde{A} be the set of unrepeated elements of A excluding the axes (the elements of the form $a(i, 0)$ y $a(0, j)$).

3.1.1. *About the signs of the elements of \tilde{A} .* Four quadrants can be distinguished as shown in Figure 3.1

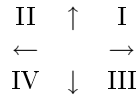


FIGURE 3.1. Quadrants of \tilde{A}

What happens to the signs of the elements of \tilde{A} from each quadrant?

Because of Theorem 3.2, we should only focus on the sign of elements of \tilde{A} originally from quadrants I, II, and IV.

- In quadrant I ($i \geq 1, j \geq 1$) all elements are positive
- In quadrant II ($i \leq -1, j \geq 1$)

It's easy to see that $j(6i + 1) \leq 0$, then $\tilde{a}(i, j) \leq 0 \forall i, j$.

¹Coordinates are in the Cartesian sense.

- In quadrant IV ($i \leq -1, j \leq -1$)
 $i + j(6i + 1) = i + j + 6ij$. ($i + j$) ≤ -1 y $ij \geq |i + j| \geq 1$. Then, the sign is positive.

Theorem 3.4. *The elements of \tilde{A} DO NOT generate prime numbers.*

Proof. $\tilde{a}(i, j) = i + j(6i + 1)$ with $i \neq 0$ and $j \neq 0$. If we put this into (2.1) and suppose p prime, then

$$p = 6\tilde{a}(i, j) + 1 = 6(i + j(6i + 1)) + 1 = 6i + 6j(6i + 1) + 1 = (6i + 1) + 6j(6i + 1) = (6i + 1)(6j + 1)$$

but since i and j are different from zero, then p would be a composite, ABS. \square

Remark 3.5. According to the signs of i and j , $6\tilde{a}(i, j) + 1$ sweeps (except for the sign) all possibilities:

- (1) If $i > 0$ y $j > 0$, the generated number is of the form $\alpha \cdot \alpha$.
- (2) If $i < 0$ y $j < 0$, the generated number is of the form $\beta \cdot \beta$.
- (3) If i y j have opposite signs, the generated number is of the form $\alpha \cdot \beta$.

See Table 4 for properties of products of α 's and β 's.

\times	α	β
α	α	β
β	β	α

TABLE 4. Table of products.

Then, summarizing previous results, we can state that for positive prime numbers:

- (1) The only even prime number 2 belongs to the sequence defined by the form $2 + 6n$ for $n = 0$. All the other even numbers in the same sequence are multiples of 2, then they are composites.
- (2) All the others even numbers that belong to sequences of the form $4 + 6n$ and $6 + 6n$, being $n = 0, 1, 2, \dots$, are multiples of 2 and obviously they are composites.
- (3) The first odd prime number 3 belongs to the sequence defined by the form $3 + 6n$ for $n = 0$. All the other odd numbers in the same sequence are multiples of 3, then they are composites.
- (4) Then, except 2 and 3, all the other prime numbers belong to classes α ($1 + 6n$) or β ($5 + 6n$).
- (5) Therefore, taking into account Table of Products $\alpha \times \alpha = \alpha$, $\beta \times \beta$, and $\alpha \times \beta = \beta$, all composite numbers generated by these products belong to classes α and β .
- (6) Then all α prime numbers are those expressed by $(1 + 6n)$ for $n = 0, 1, 2, \dots$, except:
 - (a) Number 1 expressed by $1 + 6n$, being $n = 0$.
 - (b) All even and odd powers of all class α numbers, as $(1 + 6n)^{2+k}$, for $n = 1, 2, 3, \dots$, and $k = 0, 1, 2, \dots$
 - (c) All products between β numbers and all their even and odd powers, as: $[(5 + 6n)(5 + 6(n + m))]^{1+k}$, for $n = 0, 1, 2, \dots$, $m = 0, 1, 2, \dots$, and $k = 0, 1, 2, \dots$
- (7) All β prime numbers, except all the products of each β number with all α numbers, except 1, like $(5 + 6n)(1 + 6m)$, for $n = 0, 1, 2, \dots$, and $m = 1, 2, 3, 4, \dots$

4. LEOPOLDO'S THEOREM AND THE π FUNCTION

4.1. Leopoldo's Theorem. We defined the \tilde{A} set as a list of all the non repeated off-axis elements of A . A simple expansion showed that the elements of \tilde{A} do not generate prime numbers by (2.1). Finally, we stated and proved Leopoldo's theorem [1]:

Theorem 4.1. (*Leopoldo's Theorem*²) $G_\alpha = \mathbb{Z} - \tilde{A}$.

Proof. Every prime number of absolute value greater than 3 is either α or β , and products between those classes of equivalence are closed on themselves. Because of Theorem 3.4 and Remark 3.5, we know that \tilde{A} generates all possible α and β composite numbers, and so the elements n of $\mathbb{Z} \setminus \tilde{A}$ generate prime numbers (except for the sing) by $6n + 1$. \square

This means that all integers not generated by (3.1) will generate all primes of absolute value greater than 3 by (2.1) (and thus, this works as a sieve).

In order to see the whole picture we are going to suppose that instead of integers we are using real numbers for i and j . This will lead us to a graphical sight of the matrix where, keeping $a(i, j)$ as an integer, we should see hyperbolas. The plots will represent any integer c generated by the matrix, and they can be considered as level sets. These sets have certain coincidences when i and j are integers at the same time, obviously this is a point in which the elements in the matrix coincide (in space) with an element in the set of curves. Some of the curves have lots of repetitions specially when c is large, that is why the calculation of all the elements of A is futile. What we are looking here, in order to find primes, are the integers c which have integer generators i and j . This will lead us to composite numbers and indirectly to primes. Some level sets of A are shown in Figure 5.1.

Now, suppose you wish to calculate all prime numbers of absolute value greater than 3 up to a certain value $h = 6c + 1$ ($c > 0$), this means computing $\pi(h)$ using Leopoldo's Theorem. At first sight, one would have to:

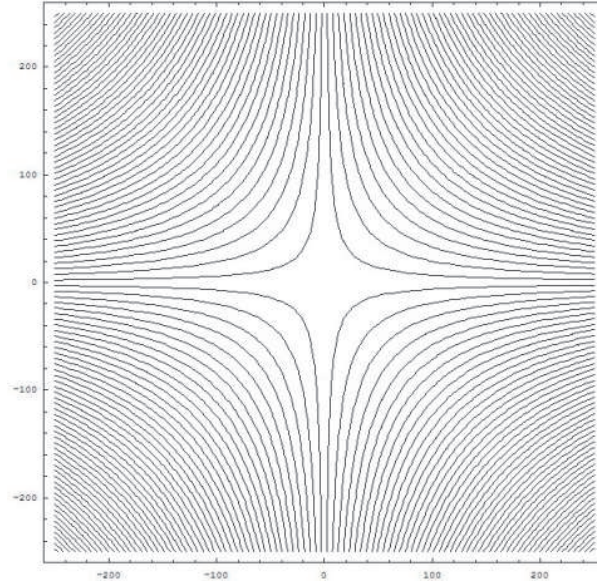
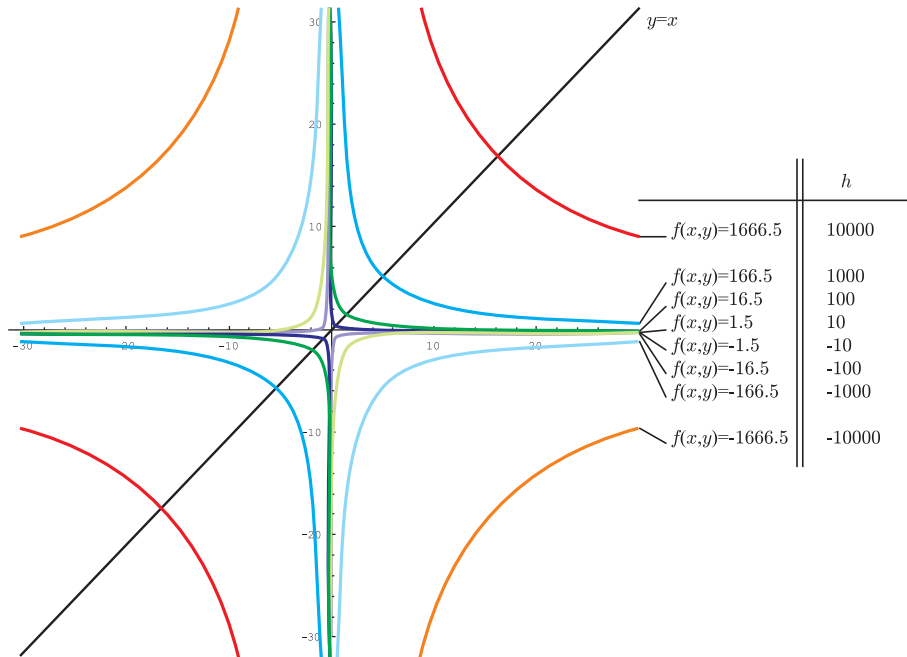
- (1) Generate all elements of \tilde{A} up to c which means that $|a(i, j)| = (h - 1) / 6$
- (2) Discard the axis elements
- (3) Sort the rest
- (4) Discard repetitions
- (5) Remove them from the interval $[-c, c]$
- (6) Count the remaining numbers
- (7) Apply (2.1) to show the primes in the interval

With large numbers, this computation would quickly become time and memory prohibitive. However, a closer look at the distribution of elements in the sieve, gives an appropriate answer to this problem.

5. LEOPOLDO'S THEOREM AND THE COMPUTATION OF THE π FUNCTION

This method is based on discarding non prime generators. In fact we only need to find two integers i and j to verify that the number c that we are checking belongs to \tilde{A} . Once we find these numbers there is no need for further search. This will simplify the problem by avoiding repetitions. In this section we are going to present an approach for an algorithmic method to compute π based in the ideas presented above.

²We named theorem 4.1 as Leopoldo's theorem because it was stated by Leopoldo Garavaglia in the Summer 2006, Spain, but the basis was not published until 2007 [5] and it was proved in [1]


 FIGURE 5.1. Several level sets of $f(x, y) = x + y(6x + 1)$.

 FIGURE 5.2. Level sets of $f(x, y) = x + y(6x + 1) = \pm c$.

In Figure 5.2, we show the representation of the level sets $f(x, y) = \pm c$ ($c > 0$). We must only consider non-repeated elements of \tilde{A} originally from within the “star” delimited by

$$(5.1) \quad f(x, y) = \pm c = \pm (h - 1) / 6$$

5.1. An algorithmic approach. An exploration of the level sets allows algorithmic approach to $\pi(h)$.

Algorithm 5.1. *We define the Λ algorithm of arguments c_1 and c_2 ($c_2 > c_1 \geq 8$), as the procedure that*

- (1) Declares a natural variable $L = 0$
- (2) For c taking every integer value from c_1 to c_2
 - (a) For integers x from $x = -\lfloor (c+1)/5 \rfloor$ to $x = -\lfloor (\sqrt{1+6c}+1)/6 \rfloor$ verifies if
$$\frac{c-x}{6x+1}$$
takes an integer value³.
 - (i) if it finds one, adds 1 to L and goes to step (2)(c)
 - (ii) if it doesn't find any, goes to step (2)(b)
 - (b) For integers x from $x = 1$ to $x = \lfloor (\sqrt{1+6c}-1)/6 \rfloor$ verifies if
$$\frac{c-x}{6x+1}$$
takes an integer value.
 - (i) if it finds one, adds 1 to L and goes to step (2)(c)
 - (ii) if it doesn't, prints c and $6c+1$, and goes to step (2)(c)
 - (c) For integers from $x = -\lfloor (c+1)/7 \rfloor$ to $x = -1$ verifies if
$$\frac{-c-x}{6x+1}$$
takes integer values.
 - (i) if it finds one, adds 1 to L and goes to the next value of c .
 - (ii) if it doesn't find any, prints $-c$ and $-6c+1$, and goes to the next value of c .
- (3) Once the process has been completed up to c_2 , reports the accumulated value of L :

$$\Lambda(c_1, c_2) = L_{final}$$

This algorithm tells the amount of numbers that *do not* generate prime numbers by $6c+1$ in the interval $[c_1, c_2]$, and also gives the list of the missing values as well as the primes generated by them. So, the amount of prime numbers between $h_1 = 6c_1 - 1$ and $h_2 = 6c_2 + 1$ ($c_1 > 8$) is:

$$(5.2) \quad \Delta\pi = 2(c_2 - c_1) - \Lambda(c_1, c_2) + 1$$

In the particular case of $c_1 = 8$ ($h_1 = 47$) and $h = 6c_2 + 1$:

$$(5.3) \quad \pi(h) = 2c_2 - \Lambda(8, c_2)$$

6. RESULTS AND FUTURE WORKS

In Table 5, it may be seen that the the proposed algorithm for the π function agrees with known results for several testing values. Equations (5.2) and (5.3) enable a parallelization of the computation of the π function with the Λ algorithm.

Calculation time grows with the parameter c . The last value in the table took nearly an hour to be computed with an AMD Athlon 64 X2 Dual Core 4200+. All values were calculated using (5.3). The memory used remains stable, since it doesn't take more than it needs to store the variables and operations in steps (2)(a), (2)(b) and (2)(c).

³This means, $(c-x) \equiv 0 \pmod{6x+1}$.

h	$\pi(h)$ (Mathematica 5)	$\pi(h)$ (Λ Algorithm)	$\pi(h-3)$ [4]
$10^2 + 3$	27	27	25
$10^3 + 3$	168	168	168
$10^4 + 3$	1229	1229	1229
$10^5 + 3$	9593	9593	9592
$10^6 + 3$	78499	78499	78498
$10^7 + 3$	664579	664579	664579

TABLE 5. Λ algorithm based results versus several known values of $\pi(h)$.

Given the fact that this algorithm evaluates all values between c_1 and c_2 , as well as the independence of the obtained value $\Delta\pi$ with other intervals, a parallel expression of the Algorithm is possible using n independent processors where the range of values c for each acting processor is

$$C_{1P} = c_1 + \left(\frac{c_2 - c_1}{\#P} \right) P_n$$

$$C_{2P} = C_{1P} + \left(\frac{c_2 - c_1}{\#P} \right) P_n$$

and so on, where $\#P$ is the total number of processors and P_n is the number of a given processor in the cluster.

Since the computing load is equally divided between computing nodes, the efficiency of the process asymptotically approaches 1 (each computing node uses nearly 100% of itself to calculate). The speedup tends to $\#P$ as new computing nodes are added whenever $(c_2 - c_1) \gg \#P$. In a future paper we will deal with the case $(c_2 - c_1) \leq \#P$

7. CONCLUSIONS

We have seen that using a simple classification of numbers we can group primes in families. We have also seen that primes and its multiples are grouped in the same classes according to the table of products 4. This classification leads us to the separation of composite numbers among primes, and the characteristic index number that do not generate primes. The matrix A which is the matrix of indexes (without the axis) and we use it in order to find primes. Also by analyzing the matrix as continuous hyperbolas we have found a way to count the amount of elements of A from a set of integers. In this way by counting the amount of “non prime generating indexes” we can obtain indirectly the amount of primes in a given section. The advantage of this method is that we can calculate the amount of primes by section. That means that this is not only an accumulative method, we can use it in any section of natural numbers and also use various computers to process different sections at the same time. Finally we can also use known results of $\pi(h)$ and start counting primes from the last well known result. As mentioned above, in future works we would like to analyze the use of several computers in order to find $\pi(h)$ and test the proposed algorithm.

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